# A Study of Heat and Mass Transfer of Magnetohydrodynamic Fluid Flow Problems

**Research plan**

**For Registration in Ph. D. Program**

**Submitted by**

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**Under the Guidance of**

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# ANNEXURE- A

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11) **Percentage of Marks / CGPA Obtained:** 66.4%

12) **Title of the Research Plan:** A Study of Heat and Mass Transfer of Magnetohydrodynamic Fluid Flow Problems

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# CERTIFICATE

This is to certify that this research protocol entitled "**A Study of Heat and Mass Transfer of Magnetohydrodynamic Fluid Flow Problems**" by **Nimit Jain** submitted in fulfillment of the requirement for registration in Ph.D. program in **DEPARTMENT OF MATHEMATICS** under Faculty of Science of JECRC University, Jaipur during the academic year 2023, is a bonafide record work carried out under my guidance and supervision.

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# 1. Introduction

The universe contains four fundamental states of matter - solid, gas, liquid and plasma. Among these last three are collectively referred to as fluids, as they exhibit a continuous deformation under even the slightest shear stress. Fluids are classified into two categories based on their viscosity - real fluids and ideal fluids. The former exhibits viscosity, whereas the latter is assumed to be free of surface tension, compressibility, and viscosity. Therefore, a vast majority of matter in the universe can be classified as fluids.

Fluid Mechanics is the study of fluid both at motion and at rest. In modern times, it has found widespread applications in various fields, such as engineering, astrophysics, biology, metrology, physics, chemistry, geophysics, among others. Fluid Mechanics principles can be easily observed in natural phenomena such as the circulation of blood in the human body, the bird's flight in the air, the movement of fish in water etc.

Fluids can be broadly classified into two types: Newtonian and non-Newtonian.

Newtonian fluid shows the relationship among the deformation rate and shearing stress which is linear in nature. On other hand, non-Newtonian fluids show the non-linear relationship between the shearing stress and deformation rate. Non-Newtonian fluids find significant commercial applications in chemical industries like plastics and polymers, and in biological and biomedical devices that utilize their rheological properties. The extensive use of non-Newtonian fluids in the diverse fields has spurred researchers to undertake research in this area.

Scientists researching non-Newtonian fluids have proposed various models to describe their behavior. Empirical models are usually used to approximate researched data for such kind of fluids, and several such models are available.

Fluid flow can be classified into two regimes based on its characteristics. Laminar flow is a structured flow which has smooth and organized motion of fluid particles in well defined or laminar layers. This flow typically occurs at relatively small Reynolds numbers. On the other hand, turbulent flow is characterized by agitated and unstable flow due to random directional motion of fluid particles in the unorganized manner, typically occurring at higher Reynolds numbers. Based

on the flow structure viscous flows are classified as turbulent or laminar. The laminar regime flow structure is characterized by smooth motion in layers. In contrast, in turbulent regime, the flow structure is characterized by fluid particles random and three dimensional motions.

Fluids are categorized either gases or liquids, with some fluids having the ability to conduct electricity, such as ionized gas or mercury. The electrical conductivity of these fluids only becomes relevant when they move in an electromagnetic field, otherwise, they are considered as ordinary fluids. Magnetofluiddynamics (MFD) is the study of motion in the presence of magnetic field in the electrically conducting fluid. When the fluids are incompressible, like mercury, and maintains constant properties like thermal conductivity, viscosity, and electrical conductivity, the study is referred to as magnetohydrodynamics (MHD) or hydromagnetics.

Physical phenomena of mathematical models, including fluid flow, often lead to differential equations. The equations may be partial or ordinary differential equations. Solving these equations is necessary for studying real-world situation, but it can be challenging since the solutions are functions of independent variables. Even seemingly simple differential equations can be difficult to solve, and there is no guaranteed method for solving them.

The similarity method involves searching for specific transformations that can get ordinary differential equation from partial differential equation. This method is used to reduce the independent variables appearing in the changed equation compared to the original equation. A similarity transformation of a partial differential equation involves changing the independent and dependent variables in the equation. The goal is to obtain an equation with fewer independent variables, at least one less than the original equation. For instance, if a partial differential equation with two independent variables, the similarity transformation will convert it into an ordinary differential equation.

Reynolds, Blasius, Buckingham, and Sedov made significant contributions to the development of similarity parameters for physical sciences. However, it was not until Morgan's work in 1952, building upon Birkhoff's concepts from 1950, which demonstrates the invariant nature of the boundary layer equations under the specific group of transformations by reducing the independent variables in a series. In the literature, the similarity methods like dimensional analysis technique, group theoretic method, the separation of variables and free parameter method are widely used.

## 1.1 Governing equations of fluid

### 1.1.1 Equation of State

State variables solely rely on the system's state. In thermodynamics, variables of the primary state are denoted by $T$ , $p$ and $\rho$ , denoting $p$ = pressure, $\rho$ = density and $T$ = temperature of fluid. These variables are related by the equation $F(p, \rho, T) = 0$ .

### 1.1.2 Equation of Continuity

The concept of mass conservation, as described in the equation of continuity, posits that the rate at which mass accumulates within a specific volume depends on the difference between the mass flowing into that volume and the rate of mass generation within it. Mathematically, this equation can be represented as follows:

$$\frac{D\rho}{Dt} + \rho \nabla . \vec{V} = 0$$

### 1.1.3 Equations of Motion

The concept of preserving linear momentum finds expression in the equation of motion. These equations are founded upon Newton's second law, which asserts that the cumulative force acting on a system equates to the rate at which its linear momentum changes. The Navier-Stokes equations serve as a means to describe the movement of fluids. In situations involving a viscous and compressible fluid, it is possible to articulate the Navier-Stokes equations as follows:

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \rho f_x$$

### 1.1.4 Equation of Energy

The conservation of energy is expressed by the equation of energy. In the case of viscous compressible fluids, this equation can be used and applied.

$$\rho \frac{D}{Dt}\left(e + \frac{\mathrm{V}^2}{2}\right) = \rho q + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) - \frac{\partial(up)}{\partial x} - \frac{\partial(vp)}{\partial y}$$

$$- \frac{\partial(wp)}{\partial z} + \frac{\partial(u\tau_{xx})}{\partial x} + \frac{\partial\left(u\tau_{yx}\right)}{\partial y} + \frac{\partial(u\tau_{zx})}{\partial z} + \frac{\partial\left(v\tau_{xy}\right)}{\partial x} + \frac{\partial\left(v\tau_{yy}\right)}{\partial y}$$

$$+ \frac{\partial\left(v\tau_{zy}\right)}{\partial z} + \frac{\partial(w\tau_{xz})}{\partial x} + \frac{\partial\left(w\tau_{yx}\right)}{\partial y} + \frac{\partial(w\tau_{zz})}{\partial z} + pf.V$$

## 1.2 Boundary layer flow

The development of boundary layer theory was a pivotal moment in understanding fluid friction, especially at high Reynolds numbers. In 1909, Prandtl introduced the concept of the boundary layer, simplifying the Navier-Stokes equations into what we now call Prandtl's boundary layer equations. When physical effects are confined to small regions of solid boundaries in that conditon the boundary layer phenomena becomes prominent and it happens due to the non dimensional diffussion parameters like Reynolds number, magnetic Reynolds number. Due to this theory only we can divide the flow into two regions:

Boundary Layer Region: This is an extremely thin layer near the solid body, known as boundary layer, where viscous effects dominate.

Outer Region: Beyond boundary layer, viscous effects become negligible, and the fluid is treated as inviscid. This distinction helps explain the significance of viscosity in calculating frictional drag and flow separation.

The boundary layer equations, while simpler than the general Navier-Stokes equations, remain non-linear partial differential equations. They are most easily handled mathematically when dealing with uncomplicated geometries. These equations pertain to the velocity, thermal, or magnetic boundary layers, with the thickness of these boundary layers being inversely related to the square root of the respective diffusion number.

## 1.3 Magnetohydrodynamic (MHD) flow

Hannes Alfvén (1908-1995) is credited with the discovery of hydromagnetic (MHD) as a field which deals with the study of the behavior of electrically conductive fluids in magnetic fields. MHD, also known as magneto-fluid-dynamics or hydromagnetics, specifically focuses on the magnetic properties and behavior of such fluids. This field is applicable to conductive fluids like liquid metals, electrolytes, saltwater, and plasmas, where the fluid is considered to be incompressible, and properties like thermal conductivity, viscosity, and electrical conductivity are assumed to be constant.

## 1.4 Some important types of fluids

**1.4.1 Casson Fluid:** Casson, a scientist renowned for his contributions, introduced a distinctive category of non-Newtonian fluid, commonly referred to as the Casson fluid. This particular fluid exhibits shear-thinning properties, displaying infinite viscosity at zero shear rates and a threshold yield stress below which it remains immobile. Under extremely high shear rates, its viscosity approaches zero. Consequently, if the applied shear stress falls below the yield stress, the Casson fluid behaves akin to a solid. Various everyday substances, including but not limited to ketchup, jelly, salt solutions, paints, shampoo, tomato sauce, soup, concentrated fruit juices, honey, and more, exemplify Casson fluids.

**1.4.2 Jeffrey Fluid:** A Jeffrey fluid is categorized as a rheological material that belongs to the class of rate-type materials. It exhibits linear visco-elastic behavior, which finds practical applications in industries related to polymers. Examples of Jeffrey fluids include dilute polymer solutions.

The Jeffrey fluid model is particularly valuable for characterizing stress relaxation properties in non-Newtonian fluids, a feature not adequately addressed by conventional viscous fluid models. This model is especially suited for a class of non-Newtonian fluids that possess a distinct memory time scale, often referred to as the relaxation time.

**1.4.3 NanoFluid:** Nanofluid refers to a fluid in which tiny particles, each on the nanometer scale, are suspended within a base fluid, creating a colloidal solution of nanoparticles within this fluid matrix. The nanoparticles commonly employed in nanofluids are crafted from materials such

as oxides, metals, carbides, or carbon nanotubes. As for the base fluids, they encompass substances like water, ethylene glycol, and various types of oils.

**1.4.4 Williamson Fluid:** The Williamson fluid is a type of non-Newtonian fluid known for its shear-thinning behavior, where its viscosity decreases as the shear stress rate increases. This property is observed in various physiological fluids, leading to the assumption that substances like chyme in the small intestine exhibit Williamson fluid-like behavior.

Williamson fluids are valuable models for understanding the flow behavior of non-Newtonian fluids in complex biological systems. For instance, blood can be considered a Williamson fluid due to its shear-thinning behavior. In the case of blood, this property enables efficient flow through narrow capillaries and blood vessels at low shear rates while maintaining sufficient viscosity during high shear stress situations, like during rapid blood flow.

**1.4.5 MicroPolar Fluid:** Micropolar fluids are often used to describe intricate fluid systems characterized by microstructures, such as animal blood and liquid crystals. Real-world instances of micropolar fluids encompass various substances like ferrofluids, blood in motion, bubbly liquids, liquid crystals, and more, all distinguished by their inherent polar properties.

## 2. Literature Review

The concept of the boundary layer was first introduced by Prandtl (1904) in a paper addressing “fluid motion with extremely low friction”. In this seminal work, he proposed a hypothesis suggesting that in fluids with exceptionally low viscosity, the flow around a solid object could be divided into two distinct regions. The first region, said to be the boundary layer, is an extremely thin layer in close proximity to the solid surface where viscous effects dominate. The second region, which lies beyond the boundary layer, experiences negligible viscous effects and can be treated as inviscid. Subsequently, boundary layer theory was further refined and expanded upon by various researchers, including Pai (1956), Lachmann (1961), Meksyn(1961), Rosenhead (1963), Evans (1968), Schlichting (1968), and Bansal (1997) among others. Some of these scholars also delved into the theory of thermal boundary layers, exploring effects of heat transfer within boundary layer.

In a study by Kuznetsov et al. (2001), the authors explored nanofluid's convection boundary layer flow neighbouring to upright plate. Bhattacharya (2011) examined over contracting sheet that how heat sink/source affects heat and magnetohydrodynamic (MHD) flow. Nadeem et al. (2013) examined the extending sheet which permeable in nature with MHD 3D flow of Casson fluid. Sulochana et al. (2015) examined a stretching surface in occupancy of chemical reactions that how impact of non-linear thermal radiation and viscous dissipation on 3D MHD Casson fluid flow takes place. Pushpalatha et al. (2016) conducted an investigation into unstable free convection Casson fluid flow confined by movable upright flat plate in rotating system. Reza et al. (2016) discussed Casson fluid's heat transfer characteristics and boundary layer flow over non-linear extending surface which is of permeable in nature. Charan et al. (2016) analysed heat source with an upright plate along with Soret effects on MHD Casson flow. Kalyan et al. (2017) investigated hydromagnetic Casson fluid flow in middle of two parallel plates in porous space, considering thermal radiation and chemical reaction. In existence of heat absorption and thermal radiation, Omokhuale et al.(2019) investigated an electrically flowing fluid while taking into account a plate with a uniformly applied cross-sectional magnetic field. Suresh et al. (2020) considered viscous dissipation and radiation effects by examining Casson fluid's steady MHD flow in inclined porous sheet. Deka (2020) investigated infinite upright plate inserted in porous medium through time dependent MHD convection free flow of a Casson fluid. Non-Newtonian MHD oscillatory flow of a porous media with radiation, diffusion and chemical reaction was studied by Sakthikala and Lavanya (2020). Rasheed et al. (2022) numerically elucidated impact of chemical reactions and thermal radiation with characteristics of steady 2D magnetohydrodynamic free convection Casson fluid flow over upward surface. Reddy et al. (2022) analytically investigated upward porous plate suction and permeability in a magnetic field over convection flow of Casson fluid.In a study conducted by Jain and Sharma (2022), an examination was carried out on the impact of the Soret and Dufour effects, chemical reactions, and Newtonian heating in the context of magnetohydrodynamic (MHD) flow involving a Casson fluid over a vertical porous plate, considering heat generation/absorption. Additionally, Rao and Sreenadh (2017) investigated the magnetohydrodynamic boundary layer flow over a permeable stretching/shrinking sheet situated within a porous medium. Sreenadh et al (2022), studied biological ducts, exhibiting elasticity, differ in fluid properties from pumped fluids. They analyze two-layered blood flow in arteries, emphasizing the impact of elastic parameters and viscosity ratios on velocity and flux, providing

insights into blood flow in various states. Mohammed et al. (2016), investigated mixed convection flows with nanofluids over a heated rectangular duct with baffles. In a study conducted by Nadeem et al. (2013), they examined the characteristics of two-dimensional flow involving a Williamson fluid model over a stretching sheet. Anuradha et al. (2019) delved into the analysis of magnetohydrodynamic (MHD) boundary layer flow in a steady micropolar fluid over a stretching sheet. Additionally, Fatunmbi et al. (2018) conducted research on the aspects of two-dimensional heat and mass transfer within an incompressible micropolar fluid, as it flows past a stretching surface with slip conditions. Their investigation encompassed considerations such as temperature-dependent viscosity, thermal radiation, non-uniform heat generation/absorption, and chemical reactions.

Roja et al. (2023) investigated the influence of thermal conductivity and chemical reactions on the magneto-hydrodynamic flow of a micropolar fluid over a continuously moving stretching surface with heat source/sink. The analysis took into consideration factors such as thermophoresis and thermal radiation, aiming to understand their combined effects on the system. Mahalakshmi(2023) invastigated mixed convection in a lid-driven enclosure filled with hybrid nanofluid under the influence of magnetohydrodynamics (MHD). The primary focus of this study was to examine the effects of the Richardson number ($0.01 \leq Ri \leq 100$), Hartmann number ($0 \leq Ha \leq 100$), and various combinations of hybrid nanoparticles (Ag–MgO, Ag–TiO2, and Ag–CuO) on the fluid flow and heat transfer phenomena within the enclosure. Hamza et al. (2023) explored the impact of magnetohydrodynamics (MHD) on the time-dependent mixed convection flow of an exothermic fluid within a vertical channel. The analysis took into account convective heating and incorporates Navier's slip conditions. To facilitate the analysis, the dimensional nonlinear flow equations are transformed into a dimensionless form using appropriate transformations.

Nithya et al. (2023) investigated the characteristics of steady, incompressible, 2D- hydromagnetic boundary layer flow of nanofluid over a stretched sheet, considering the influence of viscous and Ohmic dissipations. The analytical approach employed for solving the current problem is the DTM-Pade approximation method. Sadia et al. (2023)explored into the unsteady magneto-hydrodynamic flow and heat transfer, accounting for viscous dissipation, of a third-grade fluid flowing between two extensive parallel flat porous plates. Nayak (2023) examined the steady, incompressible flow of Sutterby fluid over a stretching cylinder. The analysis took into account

the influence of variable thermal conductivity, incorporating factors such as thermal slip, Darcy resistance, and sponginess. Additionally, the impact of the induced magnetic field is considered to analyze the results along the surface of the cylinder.

## 3. Justification of Selection of Research Area / Topic:

Topic of present research is "**A Study of Heat and Mass Transfer of Magnetohydrodynamic Fluid Flow Problems**".

**3.1 Importance of the problem:** Magnetohydrodynamics (MHD) is a significant field of study that focuses on the interplay between fluid flows and magnetic fields. It is a critical area of research for various fields, including engineering, geophysics, astrophysics, and environmental science. The fluid model is a non-Newtonian fluid model that incorporates the fluid's yield stress. Exploring the similarity solutions of MHD fluid flow problems is essential to comprehend the behavior of these fluids in various conditions, which can assist in enhancing the models and achieving more precise predictions.

**3.2 Research Scope:** Despite the large number of research studies on MHD fluid flows, the investigation of similarity solutions for MHD fluid flow problems is a recent development and hasn't received much attention. Therefore, there is a scope for original research in this area to bridge the research gap and contribute to the existing knowledge.

**3.3 Mathematical challenge:** Solving the governing equations of MHD fluid flow problems using similarity solutions is mathematically challenging because of their nonlinearity. Advancements in analytical and numerical techniques to solve these problems can significantly contribute to the progress of the applied mathematics.

**3.4 Relevance to current research:** MHD flows have become a topic of increased interest due to their relevance to various research areas, including plasma physics, geophysics, and fusion energy. Investigating the similarity solutions of MHD fluid flow problems can provide valuable insights into the behavior of these fluids in different situations, which could contribute to the advancement of these fields.

In summary, the selection of the research area / topic **"A Study of Heat and Mass Transfer of Magnetohydrodynamic Fluid Flow Problems"** can be justified based on its importance, research scope, mathematical challenge, and relevance to current research.

## 4. Research Objectives

**4.1 Study a mathematical model:** One of the main goals is to create a mathematical framework that accurately characterizes the behavior of MHD fluid flows. This frame work should incorporate key factors such as magnetic fields, yield stress of the fluid, and the nonlinearities inherent in the governing equations. Some non-Newtonian fluid models are as follows:

1. Power law model
2. Jeffrey fluid model
3. Herschel-Bulkley model
4. Cross-power law model
5. Bird-Carreau model

**4.2 Investigation of physical properties:** One of the research goal is to analyze the characteristics of MHD fluid flows under varying conditions, such as alterations in yield stress, fluid viscosity, and magnetic field strength. Such an investigation can lead to a better understanding of the fluid's behavior in different scenarios and the discovery of any remarkable phenomena that may occur.

**4.3 Compare with experimental results:** The goal is to assess the accuracy of the mathematical model by comparing the theoretical outcomes with experimental data. This process can provide a way to verify and improve the model, as well as identify any areas where it requires further refinement.

**4.4 Explore potential applications:** The aim is to examine the possible practical uses of the model and its solutions. This may entail studying how the behavior of MHD fluid flows

could affect different scientific and engineering fields, such as Geophysics, plasma Physics, and fusion energy.

In summary, the research objectives of **"A Study of Heat and Mass Transfer of Magnetohydrodynamic Fluid Flow Problems"** are to study a mathematical model to use similarity solutions, investigate physical properties, compare with experimental results, and explore potential applications. These objectives are grounded in the importance of the problem, the research scope, the mathematical challenge, and the relevance to current research.

## 5. Research Methodology

The proposed project is theoretical and computational one. The methodology intended to follow in my research work is based on the following components:

5.1 Study a literature survey on “Heat and mass transfer of Magnetohydrodynamic (MHD) fluid flow problems” to review existing research in the field.

5.2 Study governing equations for fluid flow and heat transfer in Newtonian and non-Newtonian fluids using fundamental conservation laws.

5.3 Implement boundary layer approximations and non-dimensional variables to balance the equations.

5.4 Use suitable transformations to convert partial differential equations into ordinary differential equations.

5.5 Solve ordinary differential equations using standard numerical methods like Runge-Kutta-Fehlberg (RKF) method, Shooting method, and Classical Perturbation Method.

5.6 Test the accuracy and robustness of methods by solving a sample problem and comparing analytical results with numerical results.

5.7 Investigate the resultant non-dimensional similarity equations through numerical methods using MATLAB and/or MATHEMATICA.

5.8 Analyze outcomes through a parametric study and compare results with existing literature to validate findings.

## 6. Expected Outcomes / Hypothesis

The expected result is to create a robust and accurate mathematical model that can describe the behavior of MHD fluid flow under various conditions. This model can be used to explore the physical properties of the fluid and its interaction with magnetic fields, as well as identify any fascinating phenomena that may emerge. The development of analytical and numerical techniques to address these problems can advance the field of applied mathematics. The study's results can have practical applications in various fields, including plasma physics, fusion energy, and geophysics. Moreover, it can lead to novel insights and comprehension of the behavior of non-Newtonian fluids, which can have significant implications for industries that involve the handling and processing of these fluids.

## 7. Work Plan/Time Lines

| **Research Activity** | | | | | | | |
|---|---|---|---|---|---|---|---|
| | **6** | **12** | **18** | **24** | **30** | **36** | ***** |
| 1.Course Work | ■ | ■ | | | | | |
| 2. Literature Survey | ■ | ■ | ■ | ■ | ■ | | |
| 3. Methodology | ■ | ■ | ■ | ■ | ■ | | |
| 4. Quarterly Reports | ■ | ■ | ■ | ■ | ■ | | |
| 5. Communication of Papers | | ■ | | ■ | ■ | | |
| 6. Preparation of Final Thesis | | | | | ■ | ■ | |
| 7. Submission of Thesis | | | | | | | ■ |

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